Structure Detection of Nonlinear Aeroelastic Systems with Application to Aeroelastic Flight Test Data: Part I

Sunil L. Kukreja and Martin J. Brenner

NASA Dryden Flight Research Center, Structural Dynamics Group, Edwards, California, USA

NASA Dryden flight Research Center Photo Collection
http://www.dfrc.nasa.gov/gallery/photo/index.html
NASA Photo: EC03–0039–1 Date: February 7, 2003 Photo By: Alan Brown
NASA Dryden's highly–modified Active Aeroelastic Wing F/A–18A shows off its form during a 360–degree aileron roll during a research flight.

NASA Dryden flight Research Center Photo Collection
http://www.dfrc.nasa.gov/gallery/photo/index.html
NASA Photo: EC03–0039–14 Date: February 7, 2003 Photo By: Alan Brown
NASA's Active Aeroelastic Wing F/A–18A research aircraft rolls upside down during a 360–degree aileron roll on a test mission.

Outline

- Introduction – Objectives
- The NARMAX Structure
- NARMAX Identification Problem
  - Order, parameter estimation & *structure detection*
- Structure Detection: Current Methods
- Results
  - Application to simulated pitch-plunge model
- Conclusions

Objectives

- Investigate applicability of NARMAX structure detection to aeroelastic systems
  - Simulated model of aircraft freeplay

- Evaluate performance
  - Experimental aeroelastic flight test data

NARMAX Model Description

- Input-output relationship:

$$y(n) = F^l[y(n-1), \cdots, y(n-n_y), u(n), \cdots, u(n-n_u), e(n-1), \cdots, e(n-n_e)] + e(n)$$

- Variety of nonlinear terms:

$$u^2(n-3) \quad u(n)u(n-1) \quad y(n-1)y(n-2) \quad u^2(n-1)y(n-2)$$

  – F^l, can also be described by hard nonlinearities such as a half-wave rectifier

- Linear-in-the-parameters
  – Linear regression techniques

Full Identification Procedure

- Model order selection
  - Determine number of input, output and error lags and nonlinearity order
- Parameter estimation
  - Determine values of unknown parameters
- Structure detection
  - Select parameters to include in model

System Order

$$y(n) = F^l[y(n-1), \cdots, y(n-n_y), u(n), \cdots, u(n-n_u), e(n-1), \cdots, e(n-n_e)] + e(n)$$

- System order represented as:

$$O \; = \; [n_u \, n_y \, n_e \, l]$$

- Output additive noise: $n_y = n_e$

$$\Longrightarrow O \; = \; [n_u \, n_y \, l]$$

Parameter Estimation

- Need an estimate of θ:

$$\min_{\theta} \frac{1}{2} |(\mathbf{Z} - \mathbf{\Psi}\boldsymbol{\theta})\|_2^2$$

  and statistics

- NARMAX models provide concise system representation
  - noise on the output enters the model as product terms with the system input and output
  - “Ordinary” least-squares $\longrightarrow$ biased: does not account for noise
- Solution extended least-squares:

$$\hat{\boldsymbol{\theta}} = (\mathbf{\Psi}^T\mathbf{\Psi})^{-1}\mathbf{\Psi}^T\mathbf{Z}; \quad \text{where} \quad \mathbf{\Psi} = [\mathbf{\Psi}_{zu}\mathbf{\Psi}_{zu\hat{\epsilon}}\mathbf{\Psi}_{\hat{\epsilon}}].$$

  - Bias addressed by modelling lagged errors
  - NARMAX formulation redefined into prediction error model, ϵ replacing e and z replacing y
  - Deterministic

Possible Terms

- NARMAX models described by few terms
- Maximum number of candidate terms:

$$
\begin{aligned}
p &= \sum_{i=1}^{l} p_i + 1; \\
p_i &= \frac{p_{i-1}(n_y + n_u + n_e + i)}{i}, \quad p_0 = 1
\end{aligned}
$$

– Example: model of order: $O = [4\ 4\ 4\ 2] \Rightarrow p = 105$ candidate terms.

Example of Candidate Terms

- Model example:

$$y(n) = y(n-1) + u(n-1) + u^2(n-1) + e(n-1) + e(n)$$

- Described by: $O = [n_u = 1 \; n_y = 1 \; l = 2] \Rightarrow p = 15$

- Candidate terms:

$$\begin{aligned} y(n) &= \theta_0 + \theta_1 u(n) + \theta_2 u(n-1) + \theta_3 u^2(n) + \theta_4 u(n)u(n-1) + \theta_5 u^2(n-1) + \theta_6 y(n-1) \\ &+ \theta_7 u(n)y(n-1) + \theta_8 u(n-1)y(n-1) + \theta_9 y^2(n-1) + \theta_{10} u(n)e(n-1) \\ &+ \theta_{11} u(n-1)e(n-1) + \theta_{12} y(n-1)e(n-1) + \theta_{13} e(n-1) + \theta_{14} e^2(n-1) + e(n) \end{aligned}$$

Structure Detection

- Select a subset of candidate terms

- Best describes output

Traditional Approaches to Structure Detection

- Covariance matrix, P_{θ}
- Stepwise Regression
- Bootstrap method

Covariance matrix, $\mathrm{P}_{\boldsymbol{\theta}}$

- T-test with regression analysis referred to as hypothesis testing: computing differences between means
- Suppose $E(\mathbf{Z}) = \hat{\theta}_1 + \hat{\theta}_2\psi_2 + \hat{\theta}_3\psi_3 + \cdots + \hat{\theta}_p\psi_p$ was fit
  - $\hat{\boldsymbol{\theta}}$'s tested against null hypothesis, $\hat{\theta}_i = 0$, $i = 1, 2, \ldots, p$
- Confidence interval for $\hat{\theta}_i$:

$$\hat{\theta}_i \pm t(\alpha/2, N-p)\hat{P}_{ii}$$

  where $\mathbf{P} = \sigma^2(\mathbf{\Psi}^T\mathbf{\Psi})^{-1}$ and $\hat{P}_{ii}$ ith diagonal element
  - t tabulated t ratio at $\alpha/2$ level of significance $(0 \le \alpha \le 1)$ with $N-p$ d.o.f.
- Significance assessed with $(1-\alpha)$% confidence that the parameter lies within this range
- Interval includes zero, indicates $\hat{\theta}_i$ is not significantly different from zero at the α level and can be removed from the model

Stepwise Regression

- Relies on the incremental change in RSS from adding or removing a parameter

- Two F distribution levels, F_{out} and F_{in}, formed to determine whether parameter(s) should be removed from the model (F_{out}) or included in the model (F_{in})
  - F-levels are based on $N - p$ d.o.f. for predetermined αth level of significance

- Statistics F_{in} and F_{out} estimated from RSS for model with p parameters as:

$$F_{\mathrm{in}} = \frac{\mathrm{RSS}_p - \mathrm{RSS}_{p+1}}{\mathrm{RSS}_{p+1}/(N-p-1)} \quad \text{and} \quad F_{\mathrm{out}} = \frac{\mathrm{RSS}_{p-1} - \mathrm{RSS}_p}{\mathrm{RSS}_p/(N-p)}.$$

- For good model parameterisations, F_{out} must not be greater than F_{in}

Bootstrap Method

- Numerical Procedure for Estimating Parameter Statistics
- Mild Conditions on Sample Errors
  - Errors independent and identically distributed (i.i.d.)
  - Zero-mean
- Application of ℓ_2 minimisation: $\rightarrow \hat{\boldsymbol{Z}}$, $\hat{\boldsymbol{\epsilon}}$ and $\hat{\boldsymbol{\theta}}$
- Assume: $\hat{\boldsymbol{\epsilon}} = [\hat{\epsilon}_1, \hat{\epsilon}_2, \ldots, \hat{\epsilon}_N]$, from Unknown Distribution, $\mathcal{F}$
  - Random resampling *with replacement*: $\hat{\boldsymbol{\epsilon}}^* = [\hat{\epsilon}_1^*, \hat{\epsilon}_2^*, \ldots, \hat{\epsilon}_N^*]$: estimates distribution $\mathcal{F}$;
    $\Longrightarrow$ Example: $N = 8$, $\hat{\boldsymbol{\epsilon}}^* = [\hat{\epsilon}_6, \hat{\epsilon}_8, \hat{\epsilon}_7, \hat{\epsilon}_3, \hat{\epsilon}_4, \hat{\epsilon}_7, \hat{\epsilon}_5, \hat{\epsilon}_1]$
- $\mathbf{Z}^* = \boldsymbol{\Psi}\hat{\boldsymbol{\theta}} + \hat{\boldsymbol{\epsilon}}^*$
  - Bootstrap ℓ_2 minimisation estimate $\hat{\boldsymbol{\theta}}^*$ computed from $\mathbf{Z}^*$
- Repeated B Times: $\hat{\boldsymbol{\Theta}}^* = \left[\hat{\boldsymbol{\theta}}_1^*, \ldots, \hat{\boldsymbol{\theta}}_B^*\right]$
- Statistics Computed from $\hat{\boldsymbol{\Theta}}^*$ at a Chosen Level of Significance, α

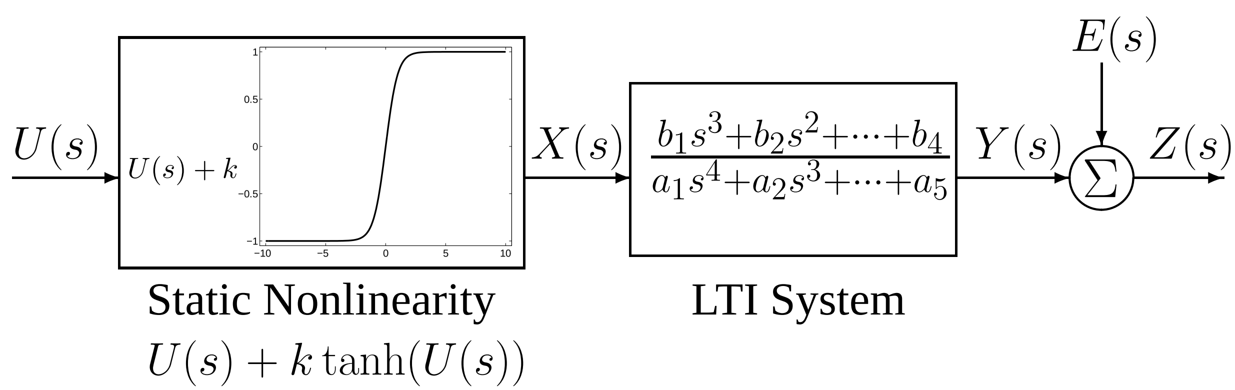
Rationale

- Structure detection provide useful process insights that can be used in subsequent development or refinement of physical models

- Investigate performance of the covariance matrix (t-test), stepwise regression and bootstrap method on a simulated NARMAX model of aeroelastic structural stiffness

- Assess performance of these techniques for applicability to experimental aircraft data.

Aeroelastic Structural Stiffness Model



- Simulated in continuous-time
- NARMAX representation aeroelastic structural stiffness model:

$$
\begin{aligned}
z(n\ ) = {} & \theta_1 z(n-1) + \theta_2 z(n-2) + \theta_3 z(n-3) + \theta_4 z(n-4) + \theta_5 u(n-1) \\
+ {} & \theta_6 u(n-2) + \theta_7 u(n-3) + \theta_8 u(n-4) + \theta_9 \tanh(u(n-1)) \\
+ {} & \theta_{10} \tanh(u(n-2)) + \theta_{11} \tanh(u(n-3)) + \theta_{12} \tanh(u(n-4)) \\
+ {} & \theta_{13} e(n-1) + \theta_{14} e(n-2) + \theta_{15} e(n-3) + \theta_{16} e(n-4) + e(n).
\end{aligned}
$$

Continuous-Time System Coefficients

- Continuous-time values correspond to those found in experiments

CT Coefficient	Value
U	18.0 m/s
a	-0.600 m
b	0.135 m
I_α	0.065 m^2Kg
k_h	2844 N/m
k_α	2.82 Nm/rad
x_a	0.247 m
c_h	27.4 Kg/s
c_α	0.180 m^2Kg/s
ρ	1.23 kg/m^3
c_{l_α}	3.28
c_{l_β}	3.36
c_{m_α}	-0.628
c_{m_β}	-0.635
k	2

Simulation Protocol

- One hundred Monte-Carlo simulations
- Each input/output realization unique
  - Inputs uniform, white, zero-mean, with variances of 8 rad^2
- Unique Gaussian, white, zero-mean, noise sequence added to output
  - Output additive noise amplitude increased in increments of 5 dB, from 20 to 5 dB SNR
- Identification data length: $N = [10,000\ 80,000]$ points increased in increments of $10,000$
- Bootstrap method $B = 100$ bootstrap replications generated to assess distribution of each parameter

....Simulation Protocol Cont.

- All three techniques parameters tested for significance at 95% confidence level

- Model posed for structure computation, additive nonlinear model:

$$z(n) = \sum_{v=1}^{q} \theta_v \psi(n) + \sum_{w=1}^{r} \theta_w f(\psi(n)) + e(n);\ q + r = p.$$

  - Model order: $O = [4\ 4\ 4\ \tanh]$
  - $\tanh$ selected as basis because a wing section response limited due to structural stiffness and appears to saturate smoothly
  - Full model description 27 candidate terms

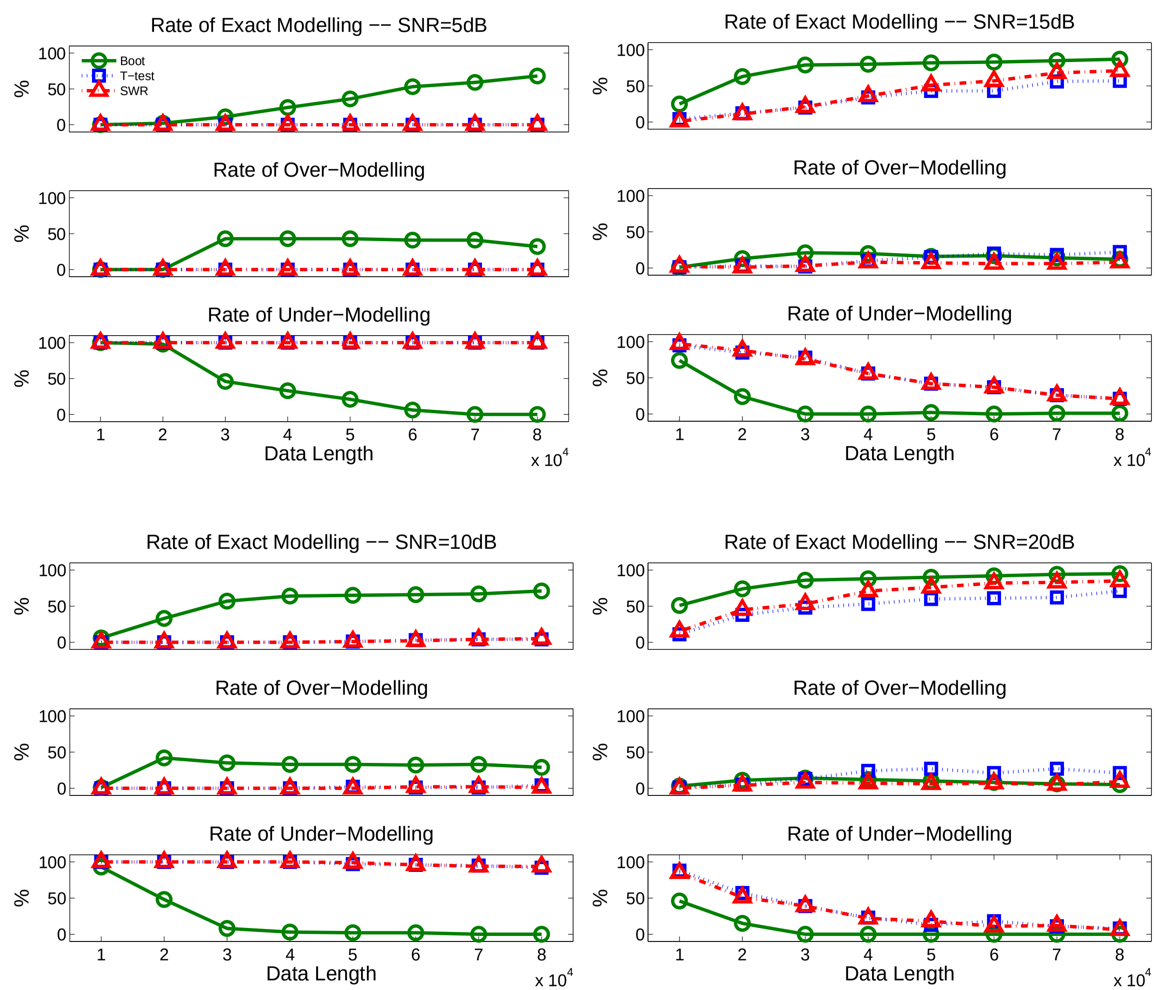
Results Classified into Three Categories

1. Exact Model: A model which contains only true system terms,

2. Over-modelled: A model with all its true system terms plus spurious parameters and

3. Under-modelled: A model without all its true system terms. An under-modelled model may contain spurious terms as well

Simulation Results



Summary of Simulation Findings

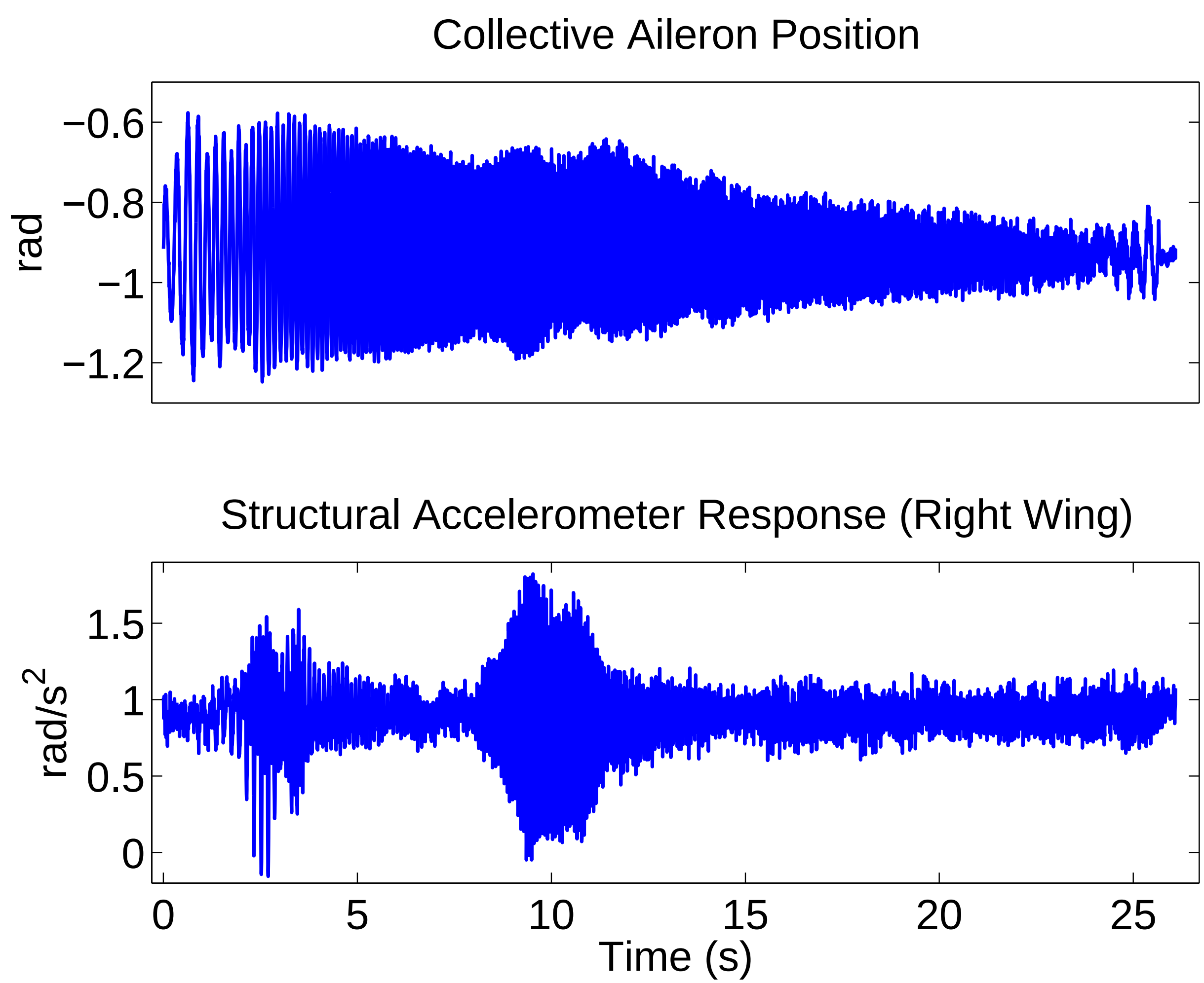
- For this over-parameterised model describing aeroelastic dynamics the bootstrap method clearly outperformed the t-test and stepwise regression

- For analysis of flight test data only implemented bootstrap structure detection method

Experimental Aircraft Data

- System identified applying bootstrap approach: model form additive non-linear & $O = [4\ 4\ 4\ \tanh]$
- Scaled hyperbolic tangent functions used because the input amplitude is less than ± 1
  - Scale factors used for the input, output and error signals in the range of $\nu = [0.1\ 1.0]$ and increased in increments of 0.1
  - A scaled hyperbolic tangent is denoted as $\tanh(\cdot, \nu)$
  - Models with every possible combination of scale factors were considered (i.e. structure computation performed on 1,000 models)
  - Full model description 27 candidate terms
- Model which yielded highest cross-validation percent fit deemed the best-fit model
  - Estimation $N_e = 5,200$: right wing & cross-validation $N_v = 5,200$: left wing

Identification Data

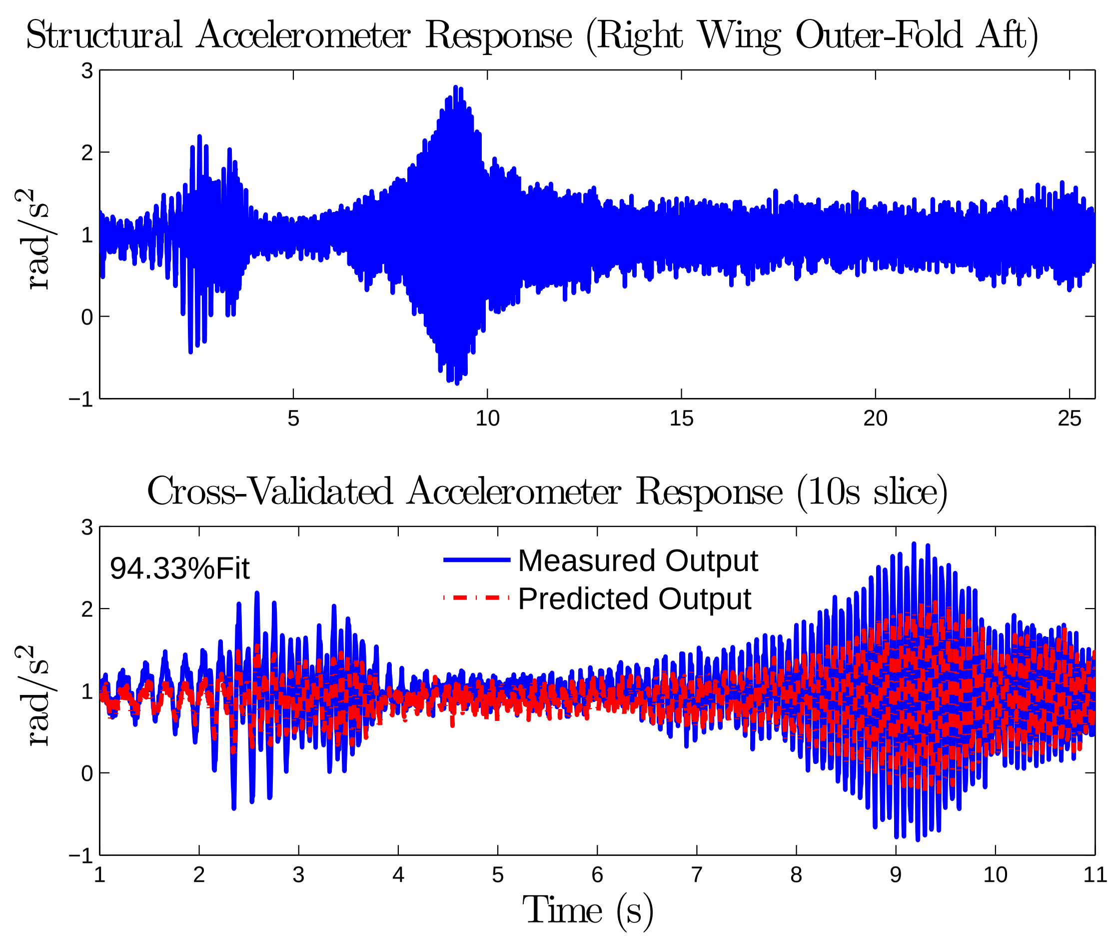


Computed Structure

- Contains 14 terms

$$
\begin{aligned}
z(n) &= \hat{\theta}_0 + \theta_1 z(n-1) + \hat{\theta}_2 z(n-3) + \hat{\theta}_3 z(n-4) + \hat{\theta}_4 u(n) + \hat{\theta}_5 u(n-2) \\
&+ \hat{\theta}_6 u(n-4) + \hat{\theta}_7 \tanh(u(n), 0.3) + \hat{\theta}_8 \tanh(u(n-1), 0.3) \\
&+ \hat{\theta}_9 \tanh(u(n-3), 0.3) + \hat{\theta}_{10} \tanh(u(n-4), 0.3) + \hat{\theta}_{11} e(n-1) \\
&+ \hat{\theta}_{12} e(n-3) + \hat{\theta}_{13} e(n-4) + e(n).
\end{aligned}
$$

Cross-Validation Data



Summary of Findings

- Simulated model of aeroelastic structural stiffness dynamics showed:
  - Given sufficient data length ($N_e = 60,000 - 80,000$), the bootstrap method had a low rate of selecting an under-modelled model (2–0%) and a high rate of selecting the exact model (60–95%) for all SNR levels
  - Both t-test and stepwise regression had difficulty computing the correct structure, with selection range of 3–70% and 2–85% respectively, for equivalent data lengths and SNR levels

- Experimental results demonstrate:
  - Bootstrap successfully reduced number of regressors posed to aircraft aeroelastic data yielding a parsimonious model structure
  - The computed parsimonious structure capable of predicting a large portion of the cross-validation data, collected on adjacent wing with different sensor
  - Suggests identified structure and parameters explain the data well

Conclusions

- Simulation results demonstrate bootstrap approach for structure computation of aircraft structural stiffness provided a high rate of true model selection
- T-test and stepwise regression methods had difficulty providing accurate results
- Work contributes to understanding of the use of structure detection for modelling and identification of aerospace systems.
- Limitation of model complexity that can be studied with these structure computation techniques
  - Result of the large number of candidate terms, for a given model order, and the data length required to guarantee convergence
  - Another approach to structure computation problem uses a least absolute shrinkage and selection operator (LASSO)

Acknowledgments

Supported by the National Academy of Sciences and the National Aeronautics and Space Administration (NASA), Dryden Flight Research Center, Aerostructures Branch (Grant Number: NASA-NASW-99027).